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Generalized Spin-Wave Theory for the Hubbard Model and D-theory Formulation

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Abstract: It is pointed out that the low energy effective theory of a generalized spin system relates to the more generalized system shown by the Hubbard-like model. When the onsite repulsion is assumed to be provided by hard-core repulsion, a generalized fermion with flavour and colour degrees of freedom is used to define the Hubbard-like Hamiltonian in this case. In the strong coupling limit and at half filling this reduces to an antiferromagnet. The *D*-theory then helps us to associate the continuum limit of the *(4+1)D* aniferromagnet to *4D* principal chiral model. It has been observed that in the strong coupling limit the problem of finding the ground state of lattice QCD is identical to that of solving the generalized antiferromagnet with Neel order playing the role of chiral symmetry breaking. In view of this, now formulate the Hubbard-like model Hamiltonian in terms of the generalized fermions with flavor and color degrees of freedom also shall consider the *D*-theoretical framework to show that the antiferromagnetic system which arises in the strong coupling limit and at half filling corresponds to the principal chiral model in the continuum limit with dimensional reduction. Also pointed out that at strong coupling and half filling the system reduces to a Heisenberg antiferromagnet. This result is analogous to the result obtained in standard Hubbard model.

Keywords: Hubbard Model, Antiferromagnet, *D*- Framework, Chiral Breaking

1. Introduction

The link beten a discretized spin system and field theory is now ll established. The framewok theory, which postulates that classical fields emerge with dimensional reduction of discrete variables, has been used by Wiese and other authors [1–4] to study this. In partic- ular it has been shown that in the continuum limit of a $(2+1)D$ quantum spin model with $SU(N_C)_L \times SU(N)_R \times U(1)_{L=R}$ symmetry is equivalent to the $2D$ principal chiral model. At zero temperature $O(3)$ symmetries of the $(2+1)D$ Heisenberg ferromagnet (antiferromagnet) breaks spontaneously giving rise to massless Goldstone bosons which are known as magnons or spin waves. Magnon's low energy effective theory is an $O(3)$ model in $2+1$ dimensions. The system experiences dimensional reduction to the $2D$ $O(3)$ model for tiny non-zero temperatures, which correspond to a finite β of the Euclidean time dimension, because the correlation of the Goldstone bosons is large in comparison to β. It is noted that dicrete spin variables undergo dimensional reduction to $2D$ if the $(2+1)D$ has massless Goldstone bosons. In this framework the $2D$ principal chiral model which is generally formulated in terms of $U(N)$ gauge fields can be represented as a system of generalized quantum spins in $(2+1)D$ [4]. In an earlier paper [5] Langmann and Semenoff have pointed out that the dynamics of Gold- stone bosons in the chiral symmetry breaking phase of the $2+1$ dimensional quantum chromo- dynamics (QCD) is identical to the effective spin wave dynamics in the 2 dimensional quantum antiferromagnet.

The fact that fermion propagation brought on by the fermion kinetic term in the lattice QCD Hamiltonian is inhibited in the strong coupling limit is the key to understanding this observation. In this case, the chiral symmetry breaking state of QCD corresponds to the Neel ordered state of the generalised antiferromagnet. Hover, the continuum limit's $3+1$ di- mensional QCD and the strong coupling limit's symmetry breaking scheme do not coincide. In reality, the chiral symmetry breaking Goldstone bosons in this limit are scalars and the mass condensate has flavour.

Hover, in spatial continuum QCD, the Goldstone bosons are pseudoscalars. The condensate is also a singlet of flavour. This implies that at least in the ak coupling continuum limit, the realistic model of the $3+1$ dimensional lattice formulation of QCD needs to be adjusted [6]. In these section shall formulate the D -theoretical framework to show that the antiferromagnetic system that arises in the strong coupling limit and at half filling corresponds to the principal chiral model in the continuum limit with dimensional reduction, also will formulate the Hubbard-like model Hamiltonian in terms of the generalised fermions with flavour and colour degrees of freedom in this section.

2. Theoretical Background

A. Hubbard-Like Model and Generalized Spin System

It has been pointed out that for a generalized spin operator in the algebra of $SU(N_F)$, N_F being the number of flavors

$$S^A = \psi_\alpha^{a\dagger} \tau_{\alpha\beta}^A \psi_a^\beta \tag{1}$$

Where $\psi_\alpha^{a\dagger}$ and ψ_a^β are the creation and destruction operators of fermion oscillators which satisfythe algebra

$$\left\{\psi_{\alpha,i}^a, \psi_{\beta,j}^{b\dagger}\right\} = \delta_{ab}\delta_{\alpha\beta}\delta_{ij} \tag{2}$$

denote the indices α, β = 1, 2,....., N_F, the flavor index, *a*, *b* = 1, 2,......., N_C color index and i, j are spatial positions. The fundamental representation of the generators τ^A of $SU(N_F)$ corresponds to Hermitian matrices which satisfy the algebra

$$\left[\tau^A, \tau^B\right] = if^{abc}\tau^C \tag{3}$$

with the normalization condition

$$Tr(\tau^A\tau^B) = \frac{1}{2}\delta^{AB} \tag{4}$$

The spin operators satisfy the algebra

$$\left[S_i^A, S_j^B\right] = if^{ABC} S_i^C \delta_{ij} \tag{5}$$

where $i(j)$ represents the spatial position of the spin. The space on which these operate is theFock space which is created by the operators $\psi_{\alpha,i}^{a\dagger}$

$$\psi_{\alpha,i}^{a\dagger}\psi_{\alpha,j}^{a\dagger}....................\left|0\right\rangle$$

operating on empty vacuum which obeys

$$\psi_{\alpha,i}^a\left|0\right\rangle = 0 \tag{6}$$

Now consider the generalized Hubbard-like model Hamiltonian

$$H = -t\Big(\sum_{i,j,\alpha,a} \psi_{\alpha,i}^{a\dagger}\psi_{\alpha,j}^a + h.c.\Big) + U\Big(\sum_{i,\alpha,\beta,a} n_{i,\alpha,a}n_{i,\beta,a} + \sum_{i,\alpha,\beta,a} n_{i,\alpha,a}n_{i,\alpha,b} + U'\sum_{i,\alpha,\beta,\gamma} n_{i,\alpha,a}n_{i,\beta,a}n_{i,\gamma,a} + \sum_{i,\alpha,a,b,c} n_{i,\alpha,a}n_{i,\alpha,b}n_{i,\alpha,c}\Big) + H.O.T \tag{7}$$

Here $H.O.T$ represent higher order terms and i, j represent the sites and t is the hopping parameter, $n_{i,\alpha,a}$ is the number of fermions at the site i with flavor index α and color index a and $U(U')$ represents the on-site repulsion which is considered here as to be caused by hard core repulsion. Now here assume that due to hard core repulsion the probability of having more than two particles at a single site is small and take into account only those terms which allow at most two particles at a single site. In view of this neglect here the terms involving the coupling U' and other higher order terms in the Hamiltonian. Using standard procedure [7, 8] now derive the corresponding $t-J$ like model in the strong coupling limit. In the low energy sector the kinetic term T_k which transfers one fermion from a singly occupied site to the empty one is given by

$$T_k = -t\sum_{i,\alpha,a}(1-n_{i,-\alpha,-a})\psi_{\alpha,i}^{a\dagger}\psi_{\alpha,i}^a(1-n_{i,-\alpha,-a}) \tag{8}$$

Where $\psi_{\alpha,i}^a(\psi_{\alpha,i}^{a\dagger})$ is the annihilation (creation) operator associated with the fermion ψ_α^a at site i. The two site term corresponding to a fermion virtually hopping from a site i, to a site j and backwards to site i can be written as

$$H^{(1)} = H_1^{(1)} + H_2^{(1)} \tag{9}$$

where

$$H_1^{(1)} = |t|^2\sum_{i,j}\sum_{\alpha,\beta,c}(1-n_{i,-\alpha,-a})\psi_{\alpha,i}^{a\dagger}\psi_{\alpha,j}^a n_{j,-\alpha,-a}n_{j,-\beta,-a}\psi_{\beta,j}^{a\dagger}\psi_{\beta,j}^a(1-n_{j,-\beta,-a}) \tag{10}$$

$$H_2^{(1)} = |t|^2\sum_{i,j}\sum_{\alpha,a,b}(1-n_{i,-\alpha,-a})\psi_{\alpha,i}^{a\dagger}\psi_{\alpha,j}^a n_{j,-\alpha,-a}n_{j,-\alpha,-b}\psi_{\alpha,j}^{b\dagger}\psi_{\alpha,j}^b(1-n_{j,-\alpha,-b}) \tag{11}$$

The term $H_1^{(1)}$ in the Hamiltonian can be expressed as

$$H_1^{(1)} = 2|t|^2 \sum_{i,j} \{\vec{S}_i.\vec{S}_j - \frac{1}{4} \sum_{\alpha,\beta,a} n_{i,\alpha,a}(1-n_{i,-\alpha,-a})n_{j,\beta,a}(1-n_{j,-\beta,-a})\} \tag{12}$$

where have taken the spin operator $\vec{S}$ at the site i

$$\vec{S}_i = \sum_{\alpha,\beta,c} \psi_{\alpha,i}^{a\dagger} \vec{\tau}_{\alpha\beta} \psi_{\beta,j}^{a} \tag{13}$$

For the term $H_2^{(1)}$ note that this involves color change which occurs through a gauge transformation

$$\psi_\alpha^a \to U_{ab} \psi_\alpha^b \tag{14}$$

The color exchange interactions which incorporate the transformations

$$\begin{aligned} \psi_{\alpha,i}^a &\to U_{ab} \psi_{\alpha,i}^b \\ \psi_{\alpha,j}^b &\to U_{ba} \psi_{\alpha,j}^a \end{aligned} \tag{15}$$

imply that the Hamiltonian is a trivial one as this involves only number operators and thus becomes irrelevant. It is noted that the three site term corresponding to a fermion virtually hopping from a site *i* to a site *j* and then to a site *k* different from the initial one. The Hamiltonian is given by

$$H^{(2)} = H_1^{(2)} + H_2^{(2)} \tag{16}$$

where

$$H_1^{(2)} = |t|^2 \sum_{\langle i,j,k \rangle} \sum_{\alpha,\beta,a,b} (1-n_{i,-\alpha,-a}) \psi_{\alpha,i}^{a\dagger} \psi_{\alpha,j}^a n_{j,-\alpha,-a} n_{j,-\beta,-a} \psi_{\beta,j}^{a\dagger} \psi_{\beta,k}^a (1-n_{k,-\beta,-a}) \tag{17}$$

$$H_2^{(2)} = \sum_{i,j,k} \sum_{\alpha,a,b} (1-n_{i,-\alpha,-a}) \psi_{\alpha,i}^{a\dagger} \psi_{\alpha,j}^a n_{j,-\alpha,-a} n_{j,-\alpha,-b} \psi_{\alpha,j}^{b\dagger} \psi_{\alpha,k}^b (1-n_{k,-\alpha,-b}) \tag{18}$$

From a similar analysis as in case of $H_2^{(1)}$ observe that the term $H_2^{(2)}$ becomes irrelevant.

So consider only $H_1^{(2)}$ for the three site term. can now construct the effective Hamilto- nian H_{eff} in the low energy sector [7] (dropping the Gutzwiller projectors which prevent any final event with more than one fermionoccupying a site). It is noted that from our above considerations the effective Hamiltonian takes the form.

$$H_{eff} = T_k + J \sum_{\langle ij \rangle} \{\vec{S}_i \vec{S}_j - \frac{1}{4} n_i n_j\} - \frac{1}{U} H_1^{(2)}\} \tag{19}$$

Here $J = 4\frac{|t|^2}{U}$. The factor 4 arises instead of 2 because the sum is over unordered pair sites. It is observed that the strong coupling limit does not exactly yield the $t-J$ like model given by

$$H_{t-J} = -t \sum_{\langle ij \rangle,\alpha,a} (\psi_{\alpha,i}^{a\dagger} \psi_{\alpha,j}^a + h.c.) + J \sum_{\langle ij \rangle} (\vec{S}_i \vec{S}_j - \frac{1}{4} n_i n_j) \tag{20}$$

as there is an extra term due to $H_1^{(2)}$ in equation (19) which is also of the order of $\frac{|t|^2}{U}$.

Hover if now impose the condition that there is one fermion per site regardless of flavor and color, this term can be avoided. In fact the occupation number of each site of the present system isgiven by

$$\langle n_{i,\alpha,a} \rangle = \frac{N_C N_F}{2} \tag{21}$$

and the condition for half filling where there is one fermion per site irrespective of flavor andcolor is attained for

$$\langle n_{i,\alpha,a} \rangle = \frac{n}{2} \text{ with } n = 1 \tag{22}$$

it is to be mentioned that in this case the kinetic term also becomes irrelevant. Thus pointed out that at strapping coupling and half filling the structure reduces to a Heisenberg antiferromagnet. This effect is analogous to the consequence obtained in standard Hubbard model form.

Now it is to be noted that the Hubbard-like model Hamiltonian has the symmetry $U(N_F) \times U(N_C)$ whereas the antiferromagnetic spin system derived from this in the strong coupling limit and at half filling has the symmetry $U(N_F)$.

Indeed in this limit the color degrees of freedom are frozen due to confinement. The hidden color degrees of freedom appear through spin fluctuation. It may be mentioned that small N_C corresponds to the quantum limit wherethe spin fluctuation becomes important. It is noted that the spin fluctuation may be represented by Q_{ij} where

$$\left\langle Q_{ij}\right\rangle = \sum_{\alpha,a}\psi^{a\dagger}_{\alpha,i}\psi^{a}_{\alpha,j} \tag{23}$$

The fluctuation consists of the phase fluctuation as ll as the amplitude fluctuation. The latter is effectively a high energy mode. Neglecting this consider only the phase fluctuationwhich is related to the local gauge transformation $\psi^{a}_{\alpha,i}$ and is given by

$$\psi^{a}_{\alpha,i} \to \psi^{a}_{\alpha,i}\exp(-i\theta_i) \tag{24}$$

associate this phase fluctuation with the gauge transformation

$$\psi^{a}_{\alpha,i} \to U_{ab}\psi^{b}_{\alpha,i} \tag{25}$$

In view of this the color gauge transformation may be considered to be caused by the phase fluctuation of the fermionic oscillator.

For an antiferromagnetic system involving these types of spin operators the classical Neel ground state is charactized by a staggered spin order parameter. Indeed in $2D$ Read and Sachdev [9] have shown that the staggered order parameter is given by

$$\mu_{\alpha\beta} = (-1)^{\sum n_i}\left(\sum_{\alpha=1}^{N_C}\psi^{a\dagger}_{a,i}\psi^{\beta}_{a,i}\right) \tag{26}$$

where n_i are certain integers related to lattice structure. For a square lattice a site i can be depicted as $i = n_1\hat{i}_x a + n_2\hat{i}_y a$ with $\hat{i}_x$ ($\hat{i}_y$) unit vector, n_i integers and a the lattice spacing. The limit $N_F \gg N_C$ is the quantum limit where fluctuations are important and the system is in a spin disordered state.

B. Formulation of the Generalizes Spin System in D-theory

Let us consider the unit vector $\vec{n}$ in terms of the spinoperator (1) so that the components of the vector $\vec{n}$ are given by

$$n^i = \frac{1}{\sqrt{k}}\psi^{a\dagger}_{\alpha}\tau^{i}_{\alpha\beta}\psi^{a}_{\beta} \tag{27}$$

With

$$n^0 = \frac{1}{\sqrt{k}}\psi^{a\dagger}_{\alpha}\tau^{0}_{\alpha\beta}\psi^{\alpha}_{\beta}$$

where

$\tau^{0}_{\alpha\beta}$ is a unit matrix and $i = 1,2..........N_F$.

Here $\frac{1}{\sqrt{k}}$ is a normalization factor so that have the relation $n_0^2 + \sum_{i=1}^{N_F} n_i^2 = 1$.

Now in 3 + 1 dimensional space-time can define the topological conserved current

$$J_\mu = \frac{1}{24\pi^2}\varepsilon_{\mu\nu\lambda\sigma}Tr(g^{-1}\partial_\nu g)(g^{-1}\partial_\lambda g)(g^{-1}\partial_\sigma g) \tag{28}$$

Where $g = n_0 I + i\vec{n}.\vec{\tau}$ with $\vec{\tau}$ being generators of $SU(N_F)$. The corresponding charge is given by

$$Q = \int J_0 d^3x \tag{29}$$

which is the winding number of the map corresponding to the homotopy group

$$\pi^3(SU(N_F)) = Z \tag{30}$$

In the continuum model of the antiferomagnetic spin system the effective action may be taken in Euclidean time dimension as

$$S = -M^2\int d^4x Tr\partial_\mu g\partial_\mu g^{-1} \tag{31}$$

with M being a constant having the dimension of mass. It may be mentioned that Radjbar- Daemi, Salam and Strathdee [10] considered discrete spin systems with a general symmetry group and shod that in the continuum limit this corresponds to nonlinear sigma model type field theories. This is consistent with the above formulation.

In this model have two types of generators such that the generators M_k rotates the vector $\vec{n}$ with N_F components to any chosen axis and the boost generators N_k which mixes n_0 with the components of $\vec{n}$. can now construct the following algebra

$$\begin{aligned}[M_i, M_j] &= i\varepsilon_{ijk}M_k\\ [M_i, M_j] &= i\varepsilon_{ijk}N_k\\ [N_i, N_j] &= i\varepsilon_{ijk}M_k\end{aligned} \tag{32}$$

This helps us to introduce the left and right generators

$$\begin{aligned}L_i &= \frac{1}{2}(M_i - N_i)\\ R_i &= \frac{1}{2}(M_i + N_i)\end{aligned} \tag{33}$$

which satisfy

$$\begin{aligned}[L_i, L_j] &= i\varepsilon_{ijk}L_k\\ [R_i, R_j] &= i\varepsilon_{ijk}R_k\\ [L_i, R_j] &= 0\end{aligned} \tag{34}$$

Thus the algebra has split into two independent subalgebras each isomorphic to $SU(N_F)$ algebra and corresponds to the group $SU(N_F)_L \otimes SU(N_F)_R$. The left and right chiral group can be taken to correspond to two mutually opposite orientations of the magnetization vector associated with the spin [11, 12].

In a lattice the action (31) can be written in the form

$$S(g) = -2M^2 \sum_{<xy>} Tr[g_x^\dagger g_y] \tag{35}$$

It can be shown that the target theory has a global $SU(N_F)_L \times SU(N_F)_R \times U(1)_{L=R}$ symmetry of the form

$$g_x \to g'_x = L g_x R^\dagger \tag{36}$$

It is noted that the fundamental degrees of freedom in the action (31) are fields represented by $N_F \times N_F$ matrices. In D-theory these fields are replaced by operators $\wedge_x^{ij}$. The D-theory Hamiltonian evolves the $4D$ system in an additional Euclidean time dimension.

By embedding the site operators in an $SU(2N_F)$ algebra it can be shown that the site operator variables transform as

$$\wedge' = \exp(-i\tau^a L^a) \wedge \exp(i\tau^b R^b) \tag{37}$$

where the τ^b are the Hermitian generators of $SU(N_F)$ [4]. This transformation implies that have the following commutation relations

$$\begin{aligned} &[L^a, \wedge^{ij}] = -\tau_{ik}^a \wedge^{kj} \\ &[R^a, \wedge^{ij}] = \wedge^{ik} \tau_{kj}^a \end{aligned} \tag{38}$$

When the operators are embedded in an $SU(2N_F)$ algebra, the $SU(N_F)_L \times SU(N_F)_R$ algebra is embedded diagonally and $\wedge^{ij}$ operators fill in the off-diagonal blocks [13-20]. In fact have the full set of commutation relations.

$$\begin{aligned} &[R^a, \wedge^{ij}] = \wedge^{ik} \tau_{kj}^a \\ &[L^a, \wedge^{ij}] = -\tau_{kj}^a \wedge^{kj} \\ &[T, \wedge^{ij}] = 2\wedge^{ik} \\ &[R^a, L^b] = [T, L^a] = [L, R^b] = 0 \end{aligned} \tag{39}$$

The operator T generates the extra U (1) symmetry. Thus the continuum limit of a $(4 + 1)D$ quantum spin model representing the antiferromagnetic system realized from the Hubbard-like model in the strong coupling limit and at half-filling has the symmetry $SU(N_F)_L \times SU(N_F)_R \times U(1)$. This is equivalent to the $4D$ principal chiral model of QCD with Euclidean time dimension.

3. Conclusion

Here, it has been conclude that the low energy effective theory of a generalised spin system relates to the more generalised system shown by the Hubbard-like model. When the on-site repulsion is assumed to be provided by hard-core repulsion, the Hubbard-like model Hamiltonian is expressed in terms of generalised fermions with flavour and colour degrees of freedom. This is reduced to an antiferromagnet at half-filling and in the strong coupling limit. The D-theory then helps us to associate the continuum limit of the $(4 + 1)D$ antiferromagnet to $4D$ principal chiral model which appears as the low energy effective theory of the quantum spin model. The chiral symmetry breaking gives rise to pseudoscalar Goldstone bosons in conformity with the $(3 + 1)D$ QCD in the continuum limit. In the ak coupling limit the dominance of the hopping term allows fermion propagation which coresponds to the color gauge interaction in the lattice QCD formulation.

4. Discussion

We have shown that the production of low energy skyrmionic excitation at the site of a fermion, which propels fermionic propagation, destroys the underlying antiferromagnetic order. Indeed, form our analysis we have note that the rearrangement of the fermionic components inside the constrained region of the bound states of the interacting system corresponds to this fermionic propagation from one place to the next in the continuum limit. As a result, a running coupling constant thatleads to asymptotic freedom is effectively created. This is equivalent to the interaction with anon-Abelian gauge field when the gauge group is $SU(N_C)$, N_C being the number of colors. This help us to the generalised spin fluctuation may be linked to the colour gauge field.

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